AR(1) Model

Sabemos que:

$$Y_t = \rho\, Y_{t-1} + \varepsilon_t$$

$$Y_{t-1} = \rho\, Y_{t-2} + \varepsilon_{t-1}$$

Entonces:

$$Y_t = \rho\, (\rho\, Y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$Y_t = \rho^2\, Y_{t-2} + \rho\, \varepsilon_{t-1} + \varepsilon_t$$

Sabemos que:

$$Y_{t-2} = \rho\, Y_{t-3} + \varepsilon_{t-2}$$

Entonces:

$$Y_t = \rho^2\, (\rho\, Y_{t-3} + \varepsilon_{t-2}) + \rho\, \varepsilon_{t-1} + \varepsilon_t$$

$$Y_t = \rho^3\, Y_{t-3} + \rho^2\, \varepsilon_{t-2} + \rho\, \varepsilon_{t-1} + \varepsilon_t$$

Si hacemos hasta N:

$$Y_t = \rho^{N+1}\, Y_{t-(N+1)} + \sum_{k=0}^{N} [\rho^k\, \varepsilon_{t-k}]$$

Si $|\rho| < 1$, entonces

$$\lim_{N \to \infty} \rho^{N+1} = 0$$

Entonces se simplifica a

$$Y_t = \sum_{k=0}^{\infty} [\rho^k\, \varepsilon_{t-k}]$$

<u>Valor esperado de Y_t</u>

$$E[Y_t] = \sum_{k=0}^{\infty} \mathrm{E}\, [\rho^k\, \varepsilon_{t-k}] = \sum_{k=0}^{\infty} \rho^k\, \mathrm{E}\, [\varepsilon_{t-k}] = 0$$

$$E[Y_t] = 0$$

<u>Varianza de u_t</u>

$$V[u_t] = V\left[\sum_{k=0}^{\infty}[\rho^k\ \varepsilon_{t-k}]\right] = \sum_{k=0}^{\infty}[\ \rho^{2k}\ V[\varepsilon_{t-k}]\ ] = \sum_{k=0}^{\infty}[\ \rho^{2k}\ \sigma^2\ ] = \sigma^2 \sum_{k=0}^{\infty}[\ \rho^{2k}\ ]$$

Recordamos la Serie Geométrica:

$$w = \sum_{k=0}^{k}[\ r^k\ ] = 1 + r + r^2 + r^3 + \cdots + r^k$$

$$r\ w = r + r^2 + r^3 + r^4 + \cdots + r^{k+1}$$

$$w - rw = 1 - r^{k+1}$$

$$w * (1 - r) = 1 - r^{k+1}$$

$$w = \frac{1 - r^{k+1}}{1 - r}$$

Notar que si $|r| < 1$:

$$\lim_{k \to \infty} \frac{1 - r^{k+1}}{1 - r} = \frac{1}{1 - r} \quad donde\ \ |r| < 1$$

$$1 + r + r^2 + r^3 + \cdots + r^{\infty} = \frac{1}{1 - r} \qquad donde\ \ \ |r| < 1$$

Por lo tanto si $|\rho| < 1$, entonces:

$$V[u_t] = \frac{\sigma^2}{1 - \rho^2}$$

Notar que en la Random Walk: $|\rho| = 1$. Si esto pasa $V[u_t] = \infty$. Sin embargo para llegar a este resultado asumimos que $|\rho| = 1$ por lo que esta interpretación no es del todo correcta.