Obtaining the $var(\hat{\alpha})$

$$Y = \hat{\alpha} + X\hat{\beta}$$

$$\bar{Y} = \hat{\alpha} + \bar{X}\hat{\beta}$$

$$var(\hat{\alpha}) = var(\bar{Y} - \bar{X}\hat{\beta}) = var(\bar{Y}) + var(\bar{X}\hat{\beta}) - 2COV(\bar{Y}, \bar{X}\hat{\beta})$$

- $Y_i = \alpha + \beta X_i + u_i$
  - $\bar{Y} = \alpha + \beta\bar{X} + \bar{u}$
    - $\bar{Y} = \hat{\bar{Y}} = \hat{\alpha} + \hat{\beta}\bar{X}$
  - $var(\bar{Y}) = var(\alpha + \beta\bar{X} + \bar{u}) = var(\bar{u})$
  - $var(\bar{Y}) = var\left(\sum \frac{u_i}{n}\right) = \frac{1}{n^2} var(\sum u_i) = \frac{1}{n^2}\sum var(u_i)$ $(no\ correlacion\ serial)$
  - $var(\bar{Y}) = \frac{1}{n^2}\sum \sigma^2 = \frac{\sigma^2}{n}$
- $var(\hat{\beta}) = \frac{\sigma^2}{\sum x_i^2}$
- $var(\bar{X}\hat{\beta}) = \frac{\bar{X}^2\sigma^2}{\sum x_i^2}$
- $COV(\bar{Y}, \bar{X}\hat{\beta}) = 0$

$$var(\hat{\alpha}) = \frac{\sigma^2}{n} + \frac{\bar{X}^2\sigma^2}{\sum x_i^2}$$

- $\bar{X}^2 = \left(\sum \frac{X_i}{N}\right)^2 = \left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right)^2 = \frac{(\sum X_i)^2}{N^2}$

$$var(\hat{\alpha}) = \frac{\sigma^2}{N} + \frac{(\sum X_i)^2}{N^2}\frac{\sigma^2}{\sum x_i^2}$$

$$var(\hat{\alpha}) = \frac{\sigma^2}{N}\left(1 + \frac{(\sum X_i)^2}{N}\frac{1}{\sum x_i^2}\right)$$

$$var(\hat{\alpha}) = \frac{\sigma^2}{N}\left(\frac{N\sum x_i^2 + (\sum X_i)^2}{N\sum x_i^2}\right)$$

$$var(\hat{\alpha}) = \frac{\sigma^2}{N}\left(\frac{N\sum x_i^2 + (\sum X_i)^2}{N\sum x_i^2}\right)$$

- $\sum x_i^2 = \sum(X_i - \bar{X})^2 = \sum X_i^2 - N\bar{X}^2$

$$var(\hat{\alpha}) = \frac{\sigma^2}{N}\left(\frac{N(\sum X_i^2 - N\bar{X}^2) + (N\bar{X})^2}{N\sum x_i^2}\right) = \frac{\sigma^2}{N}\left(\frac{N\sum X_i^2 - N^2\bar{X}^2 + N^2\bar{X}^2}{N\sum x_i^2}\right) = \frac{\sigma^2}{N}\left(\frac{\sum X_i^2}{\sum x_i^2}\right)$$